Particle number asymmetry of the early universe

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Contents; A new mechanizm generating particle number asymmetry

1. **Interaction bet. a neutral scalar of time dependent expectation value and a complex scalar which is a quantum field.**
2. **Real time evolution of particle number is obtained with non-equilibrium field theory which incorporates a density matrix specifying the initial state.**

Model of scalars with particle number violation

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} \right),$$

$$\mathcal{L}_{\text{free}} = g^{\mu\nu} \nabla_\mu \phi^\dagger \nabla_\nu \phi - m_\phi^2 |\phi|^2$$

$$+ \frac{B^2}{2}(\phi^2 + \phi^{\dagger 2}) + \left(\frac{\alpha_2}{2} \phi^2 + h.c. \right) R + \alpha_3 |\phi|^2 R$$

$$+ \frac{g^{\mu\nu}}{2} \nabla_\mu N \nabla_\mu N - \frac{M_N^2}{2} N^2$$

N **Neutral** ϕ **complex**

$$g_{\mu\nu} = (1, -a^2(x^0), -a^2(x^0), -a^2(x^0)).$$

Interaction $\mathcal{L}_{\text{int.}} = A\phi^2 N + A^*\phi^{*2} N + A_0|\phi|^2 N$

Lagrangian in terms of three real fields

$\phi_i (i = 1 \sim 3)$ ($\phi = \dfrac{\phi_1 + i\phi_2}{\sqrt{2}}, \quad \phi_3 = N$)

$$\mathcal{L} = \frac{g^{\mu\nu}}{2}(\nabla_\mu \phi_i \nabla_\nu \phi_i) - \frac{m_i^2}{2}\phi_i^2 + \sum_{ijk=1}^{3} \frac{A_{ijk}}{3}\phi_i \phi_j \phi_k$$

$m_{1,2}^2 = m_\phi^2 \mp B^2$	U(1) violation
$A_{113} = \frac{A_0}{2} + \mathrm{Re.}(A)$	$A_{223} = \frac{A_0}{2} - \mathrm{Re.}(A)$
$A_{113} - A_{223} = 2\mathrm{Re.}(A)$	U(1) violation
$A_{123} = -\mathrm{Im.}(A)$	U(1), CP violation

Initial Condition($x^0 = t_0$)

Density matrix which corresponds to mixed state.

$$\rho(t_0) = \frac{e^{-\beta H_0}}{\mathrm{Tr} e^{-\beta H_0}}, \quad \beta = \frac{1}{T} \quad (T = \text{Temperature})$$

$$H_0 = \sum_{i=1}^{3} \frac{a(t_0)^3}{2} \int d^3\mathbf{x}$$

$$\left[\pi_{\phi_i} \pi_{\phi_i} + \frac{\nabla\phi_i \cdot \nabla\phi_i}{a(t_0)^2} + \sum_{i=1}^{3} \tilde{m}_i^2 (\phi_i - v_i \delta_{i3})^2 \right].$$

v_i denotes the initial expectation value of field.

Initial($x^0 = t_0$) expectation values of fields:

$$\mathrm{Tr}(\phi_i(t_0, \mathrm{x})\rho(t_0)) = v_i \delta_{i3}$$

Initial condition for Green function:

$$\mathrm{Tr}((\phi_j(t_0, \mathrm{y}) - v_j)(\phi_i(t_0, \mathrm{x}) - v_i)\rho(t_0))$$
$$= \delta_{ij} \int \frac{d^3k}{(2\pi)^3 2\omega_i(\mathrm{k}) a(t_0)^3} \frac{\sinh \beta\omega_i(\mathrm{k})}{\cosh \beta\omega_i(\mathrm{k}) - 1} e^{-i\mathrm{k}\cdot(\mathrm{x}-\mathrm{y})}$$

Expectation value of the particle number

Particle number is related to U(1) current

$$j_\mu = \frac{1}{2}\left(\phi_2 \overleftrightarrow{\partial}_\mu \phi_1 - \phi_1 \overleftrightarrow{\partial}_\mu \phi_2\right)$$

Expectation value of 0th component of the current= particle number asymmetry/per unit volume=U(1) charge density:

$$\langle j_0(x)\rangle =$$

$$\mathrm{Re.}\left[\left(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial y^0}\right) G^{12}_{12}(x,y)\Big|_{y\to x} + \bar{\phi}_2(x)\overleftrightarrow{\partial}_0 \bar{\phi}_1(x)\right].$$

Expectation value(condensation) $\bar{\phi}(x) = \mathrm{Tr}(\phi(x)\rho(t_0))$,

Quantum part $\varphi(x) = \phi(x) - \bar{\phi}(x)$,

$G^{12}_{12}(x,y) = \mathrm{Tr}(\varphi_2(x)\varphi_1(y)\rho(t_0))$

Time ordered, anti-time ordered, non-ordered Green functions (Keldysh)

$$G^{11}_{ij}(x,y) = Tr(T(\varphi_j(y)\varphi_i(x))\rho(t_0)))$$
$$G^{22}_{ij}(x,y) = Tr(\rho(t_0)\tilde{T}(\varphi_j(y)\varphi_i(x)))$$
$$G^{12}_{ij}(x,y) = Tr(\varphi_j(y)\varphi_i(x)\rho(t_0))$$
$$G^{21}_{ij}(x,y) = G^{12}_{ji}(y,x).$$

Flow of Getting Green Function and expectation value of Field

1 Functional representation of initial density matrix $< \phi^1|\rho(t_0)|\phi^2 >$

2 Generating functional $W[J, K]$ for Green Function and Fields

$$iW[J, K] = \log \int d\phi^a$$

$$e^{i[S + c^{ab}\int d^4x J^a \phi^b + \int d^4x d^4y \phi^a(x) c^{ab} K^{bd}(x,y) c^{de} \phi^e(x)]}$$

$$S[\phi^a] = S[\phi^1] - S[\phi^2]^*$$

$$d^4x \rightarrow \sqrt{-g(x)} d^4x.$$

3 By differentiating the generating functional with respect to the source terms, one obtains:

$$\bar{\phi}^a_i(x) = \mathrm{Tr}[\phi_{\mathrm{Hi}}(\mathrm{x})\rho(\mathrm{t_0})]$$

$$= \frac{c^{ab}}{\sqrt{-g(x)}} \frac{\delta W[J,K]}{\delta J^b_i(x)}\bigg|_{J=-ij,K=-i\kappa},$$

$$\bar{\phi}^a_i(x)\bar{\phi}^e_j(y) + G^{ae}_{ij}(x,y)$$

$$= \frac{2c^{ab}}{\sqrt{-g(x)}} \frac{\delta W[J,K]}{\delta K^{bd}_{ij}(x,y)} \frac{c^{de}}{\sqrt{-g(y)}}\bigg|_{J=-ij,K=-i\kappa}.$$

$$\mathrm{Tr}[\varphi_{\mathrm{Hj}}(\mathrm{y})\varphi_{\mathrm{Hi}}(\mathrm{x})\rho(\mathrm{t_0})] = \mathrm{G^{12}_{ij}}(\mathrm{x},\mathrm{y}).$$

4 Schwinger-Dyson equation for $G^{ab}_{ij}(x,y)$ and $\bar{\phi}_i$ are obtained from 2 PI effective action $\Gamma[G,\overline{\phi}]$ which is defined by Legendre transform of W[J,K].

$$\Gamma[G,\bar{\phi},g] = S[\bar{\phi},g] + \frac{i}{2}\mathrm{TrLnG}^{-1}$$

$$+\frac{1}{2}\int d^4x d^4y \frac{\delta^2 S[\bar{\phi}]}{\delta\bar{\phi}^a_i(x)\delta\bar{\phi}^b_j(y)} G^{ab}_{ij}(x,y).$$

We write $\Gamma[G,\bar{\phi}]$up to the first order of interaction $O(A)$. Note $S_{int}[\bar{\phi}] = \dfrac{A_{ijk}}{3}\bar{\phi}_i\bar{\phi}_j\bar{\phi}_k$.

Schwinger Dyson Eqs. for $\bar{\phi}(x^0)$ and Green functions G.

$$(\Box + \tilde{m}_i^2)\bar{\phi}_i^d(x) = J_i^d(x) + \int d^4z K_{ij}^{de}(x,z)c^{ef}\sqrt{-g(z)}\bar{\phi}_j^f(z)$$

$$+c^{da}D_{abc}A_{ijk}\left\{\bar{\phi}_j^b(x)\bar{\phi}_k^c(x) + G_{jk}^{bc}(x,x)\right\},$$

$$(\vec{\Box}_x + \tilde{m}_i^2)G_{ij}^{ab}(x,y) = -i\delta_{ij}\frac{c^{ab}}{\sqrt{-g(x)}}\delta(x-y)$$

$$+2c^{ad}D_{dce}A_{ikl}\bar{\phi}_{l,x}^e G_{kj,xy}^{cb}$$

$$+\int d^4z K_{ik}^{ae}(x,z)\sqrt{-g(z)}c^{ef}G_{kj}^{fb}(z,y).$$

$D_{111} = -D_{222} = 1$. The other D_{abc} is zero.

Particle Number Asymmetry= PNA

$$
\begin{aligned}
&\langle j_0(x^0)\rangle \\
&= \frac{2}{a(x^0)^3}\hat{\varphi}_{3,t_0}\int\frac{d^3\mathrm{k}}{(2\pi)^3}\int_{t_0}^{x^0}\hat{A}_{123,t}(-\bar{K}'_{3,tt_0,0}) \\
&\Big[\Big\{\frac{1}{2\omega_{2,\mathrm{k}}(t_0)}\coth\frac{\beta\omega_{2,\mathrm{k}}(t_0)}{2} \\
&\times\Big[\dot{\bar{K}}_{1,x^0t,\mathrm{k}}\bar{K}'_{2,x^0t_0,\mathrm{k}}\bar{K}'_{2,tt_0,\mathrm{k}}-\bar{K}_{1,x^0t,\mathrm{k}}\dot{\bar{K}}'_{2,x^0t_0,\mathrm{k}}\bar{K}'_{2,tt_0,\mathrm{k}} \\
&+\omega^2_{2,\mathrm{k}}(t_0)(\dot{\bar{K}}_{1,x^0t,\mathrm{k}}\bar{K}_{2,x^0t_0,\mathrm{k}}-\bar{K}_{1,x^0t,\mathrm{k}}\dot{\bar{K}}_{2,x^0t_0,\mathrm{k}})\bar{K}_{2,tt_0,\mathrm{k}}]\Big\} \\
&-\{1\leftrightarrow 2 \text{ for lower indices}\}\Big].
\end{aligned}
$$

$\hat{\varphi}_{3,t_0}=v_3$. **For non-zero PNA $\leftrightarrow$ $v_3A_{123}\neq 0$ and $\tilde{m}_1\neq\tilde{m}_2$.**

Approximation for the time dependence of the scale factor

Keeping the first order of $H(t_0)$.

$$\frac{a(t_0)}{a(x^0)} \sim 1-(x^0-t_0)H(t_0), \quad H(t_0)=\frac{\dot{a}}{a}\Big|_{x^0=t_0}$$

$$\begin{aligned}\hat{A}_{123}(t) &= A_{123}\left(\frac{a(t_0)}{a(t)}\right)^{\frac{3}{2}} \\ &= A_{123}(1-\frac{3}{2}(t-t_0)H(t_0)).\end{aligned}$$

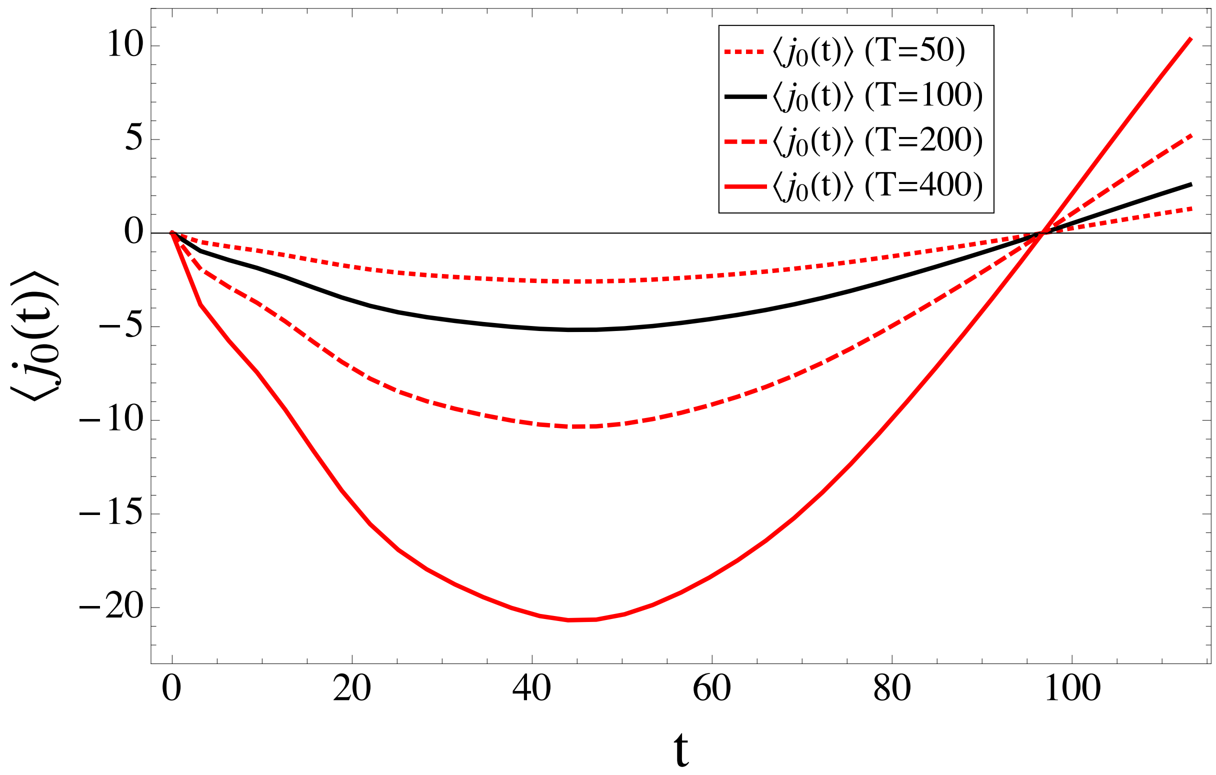

Figure 1: T **dependence.** $t = 0.35(x^0 - t_0)$ $(\tilde{m}_1, \tilde{m}_2, B, H_{t_0}, \omega_{3,0})=(0.04, 0.05, 0.02, 10^{-3}, 0.0035)$.

- **Fig.1 The amplitude of PNA increases as T increases.**
- **Fig.2 B dependence. As $2B^2 = \tilde{m_2}^2 - \tilde{m_1}^2$ increases, the amplitude also increases and the period of the oscillation becomes shorter.**

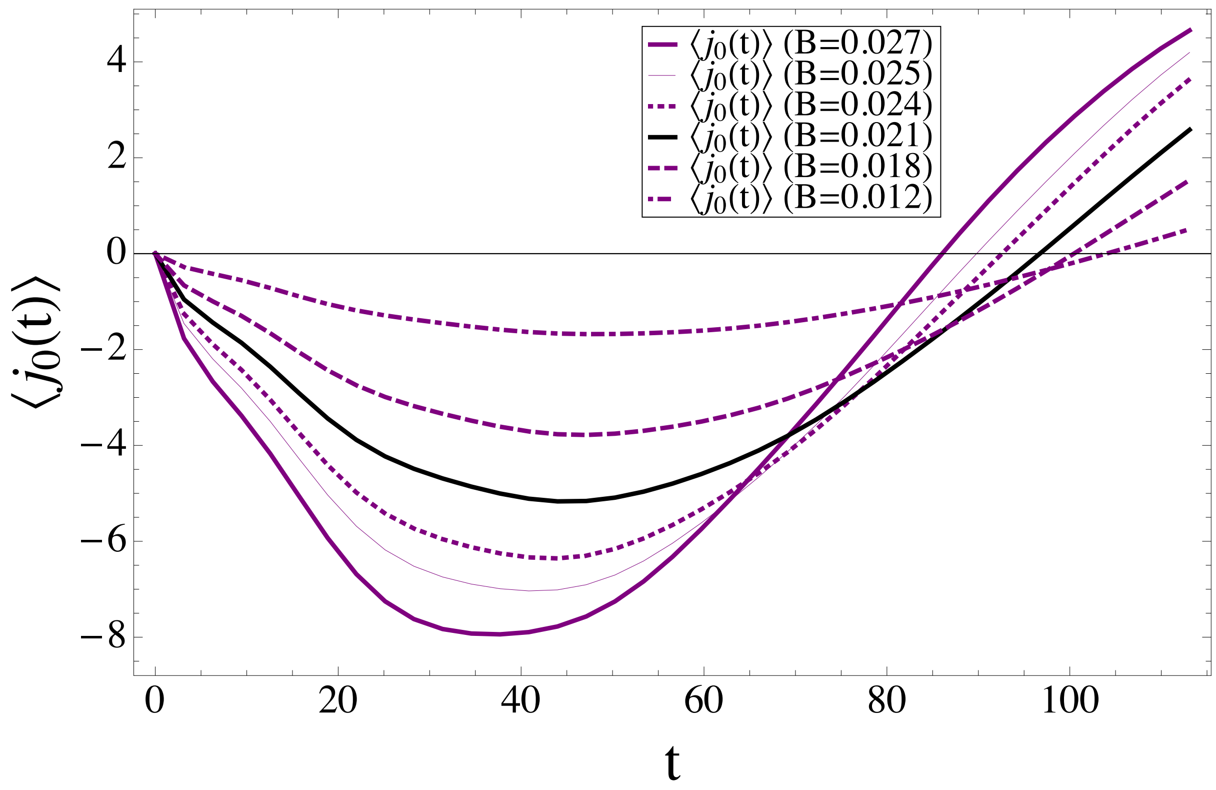

Figure 2: B **dependence.** $t = 0.35(x^0 - t_0)$. $(\tilde{m}_2, T, H_{t_0}, \omega_{3,0}) = (0.05, 100, 10^{-3}, 0.0035)$.

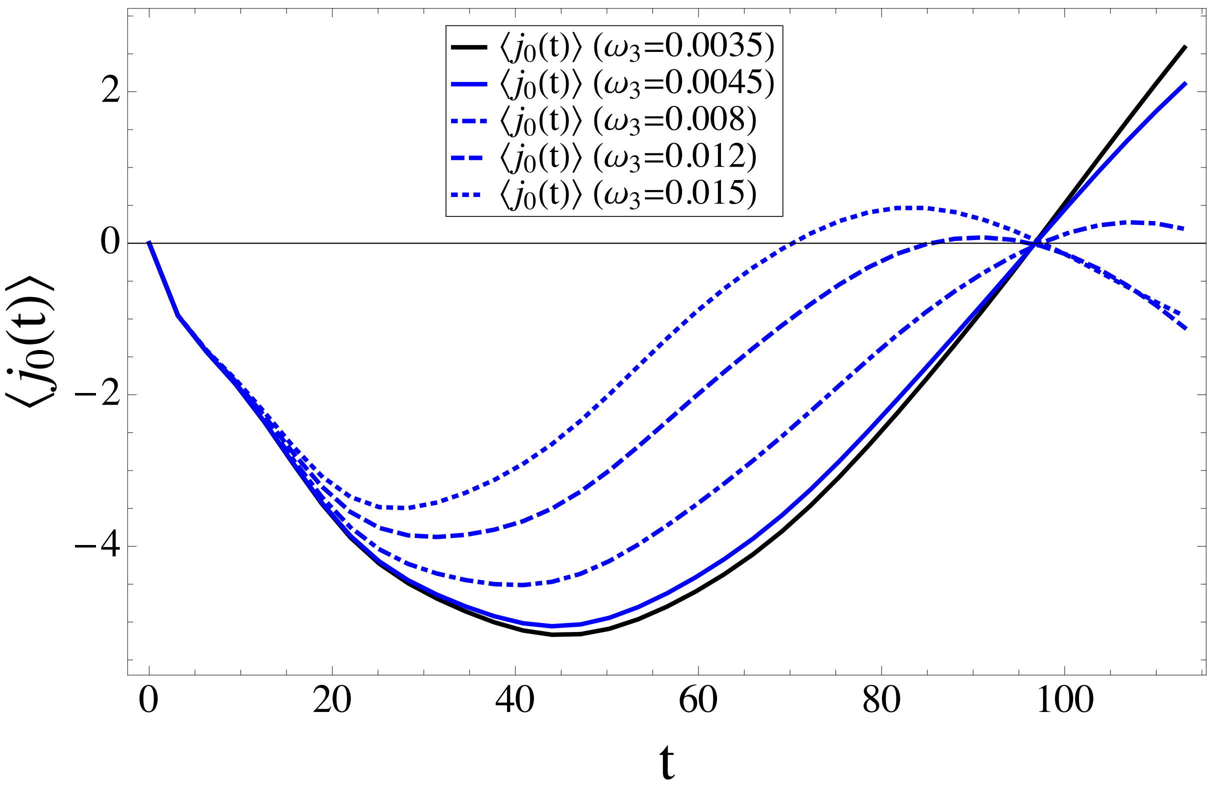

Figure 3: The $\omega_{3,0}$ dependence. $t = 0.35(x^0 - t_0)$.$(\tilde{m}_1, \tilde{m}_2, B, T, H_{t_0})$=$(0.04, 0.05, 0.021, 100, 10^{-3})$.

Fig.3 dependence on $\omega_{3,0}$(The angular frequency of the expectation value). $\varphi_3(x^0) = v_3 \cos(\omega_{3,0}(x^0 - t_0))$. When $\omega_{3,0}$ becomes larger than $\tilde{m}_2 - \tilde{m}_1$, the period becomes shorter.

Fig.4 $H(t_0)$ dependence. As $H(t_0)$ increases, the density(PNA) decreases as the dilution due to the expansion of the volume becomes significant.

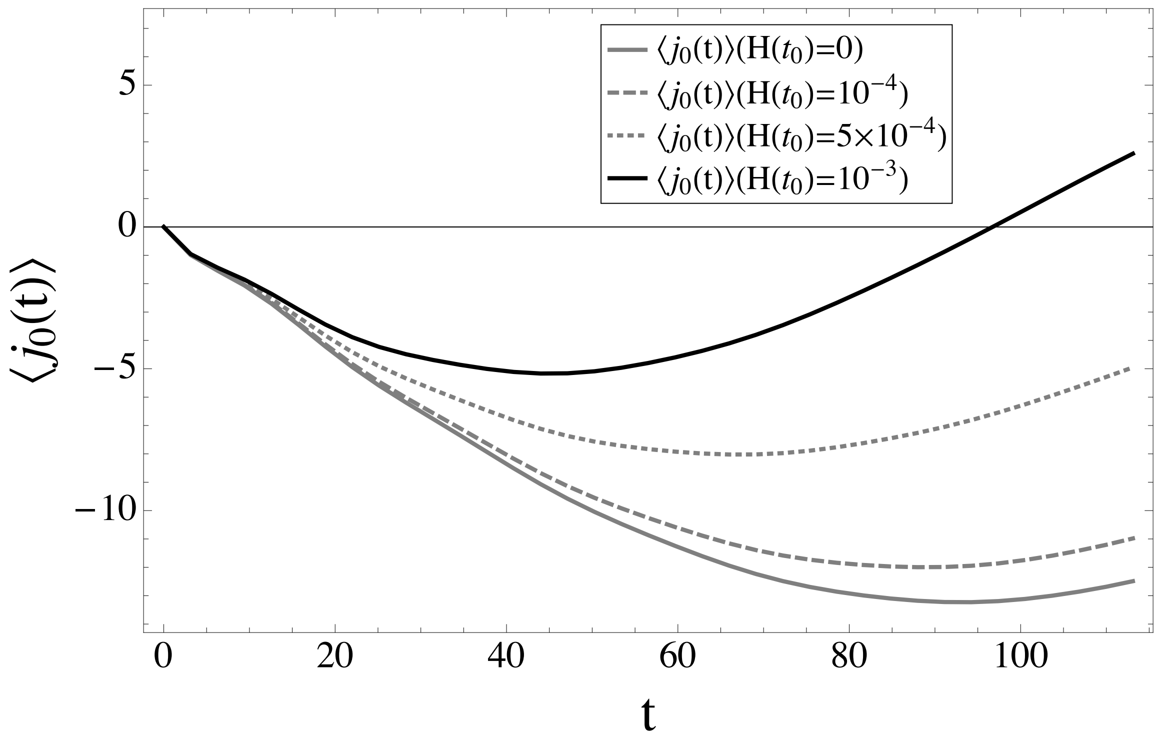

Figure 4: H_{t_0} **dependence.** $t = 0.35(x^0 - t_0)$ $(\tilde{m}_1, \tilde{m}_2, B, T, \omega_{3,0})=(0.04, 0.05, 0.021, 100, 0.0035)$.

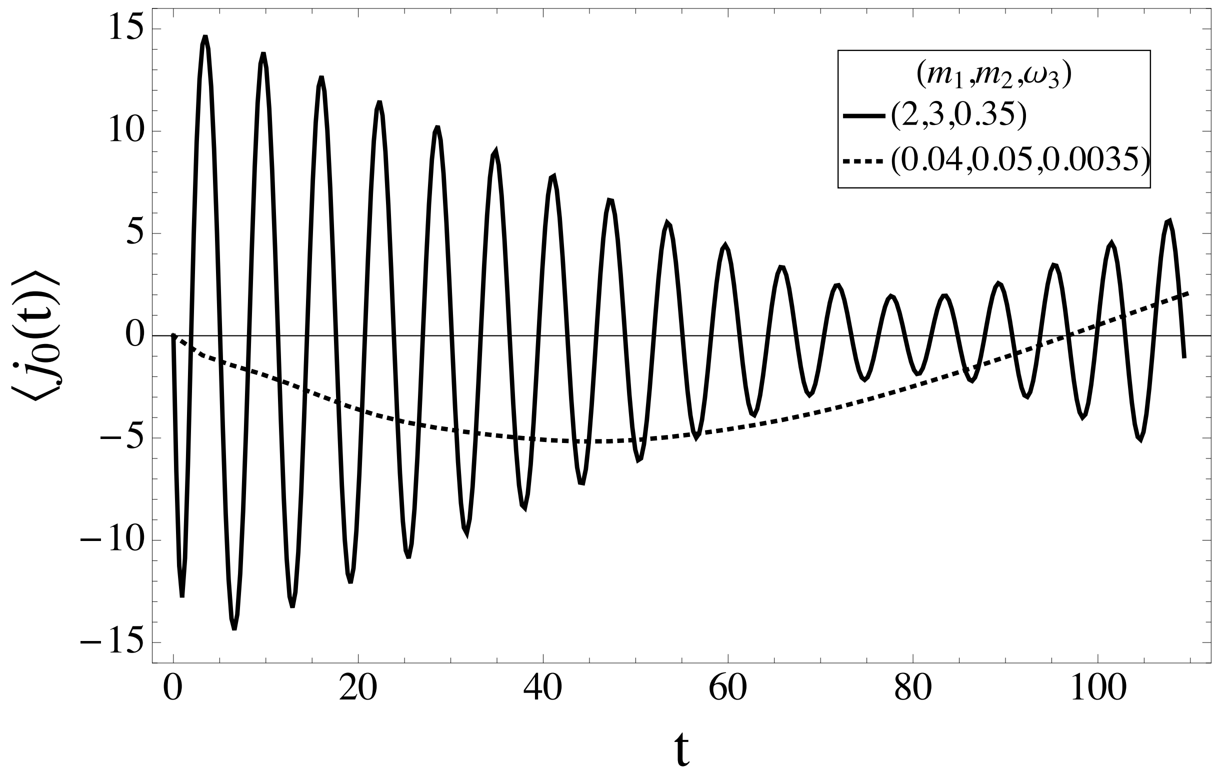

Figure 5: $t = 0.35(x^0 - t_0)$. $(T, H_{t_0}) = (100, 10^{-3})$. **The black (dotted)lines correspond to** $B = 1.58(B = 0.021)$.

We interprete the simulation in Fig.5 (the longer period case) as realistic situation by assigning the unit in (GeV) as follows.

Assumption:

The universe around at $x^0 = t_0$ is radiation dominated era with $g^* \simeq 100$.

$$10^{-5} = \frac{H(t_0)}{T} = \frac{\pi}{3}\sqrt{\frac{4\pi g^*}{5}}\frac{T(GeV)}{M_{pl}(GeV)}.$$

$$T(GeV) = 10^{13}(GeV), H(t_0) = 10^{-5}T = 10^{8}(GeV).$$

For the longer period case in Fig.5, where $T = 10^{13}(GeV)$, the ratio of PNA and the entropy at $t = 50$, is given by,

$$\frac{< j_0(t=50) >}{s} = \frac{-5 \times 10^{11}(GeV)}{T(GeV)} \frac{A_{123}}{T} \frac{v_3}{T} \frac{45}{2\pi^2 g^*}$$

$$= -1 \times 10^{-11} \frac{A_{123}(GeV)}{10^8(GeV)} \frac{v_3(GeV)}{10^{10}(GeV)}$$

$t = 50 \rightarrow 1.5 \times 10^{-9}$(1/GeV)$= 10^{-33}$(sec).

Summary

- We propose that the oscillating neutral scalar's expectation value couples with a complex scalar and the interaction generates the Particle Number Asymmetry (PNA).
- We study the real time evolution of PNA.
- The amplitude of PNA is proportional to CP violating coupling A_{123} and mass difference of two scalars $\tilde{m}_2 - \tilde{m}_1$ which originally form a complex scalar.
- The third condition of Sakharov is not satisfied yet since the Hubble time seems to be longer than the period of oscillation of PNA, typical time scale of particle number violating interaction.